Stochastic Optimization for Coupled Tensor Decomposition with Applications in Statistical Learning

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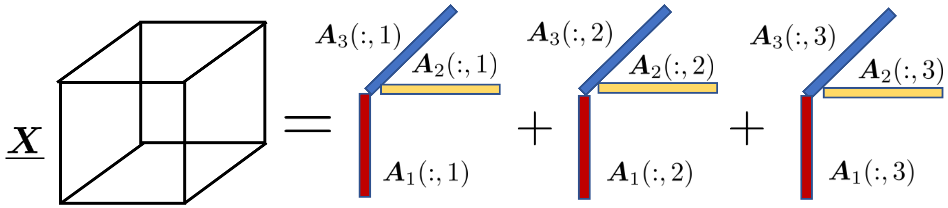
Outline

- **Motivation.**
- **Problem Modeling.**
- **Existing Approaches.**
- **Proposed Algorithm.**
- **Experimental Results.**
- **Conclusion.**

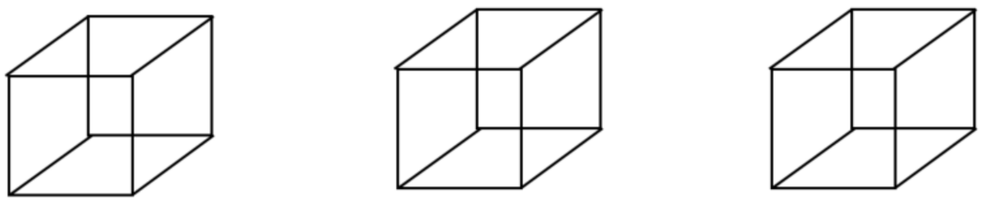
Coupled Tensor Decomposition

- *Canonical Polyadic Decomposition (CPD)* aims at factoring an K-way tensor $\underline{\boldsymbol{X}}$ with rank F to its latent factors, represented as $\underline{\boldsymbol{X}} = [\![\boldsymbol{A}_1, \ldots, \boldsymbol{A}_K]\!]$.



3-way tensor $\underline{\boldsymbol{X}}$ with $F = 3$, $\underline{\boldsymbol{X}} = [\![\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3]\!]$.

- *Coupled Tensor Decomposition* simultaneously performs CPD to a number of tensors $\underline{\boldsymbol{X}}_1, \ldots, \underline{\boldsymbol{X}}_N$ that share some of the latent factors.



$\underline{\boldsymbol{X}}_1 = [\![\boldsymbol{A}_1, \boldsymbol{A}_2, \boxed{\boldsymbol{A}_3}]\!]$ $\quad$ $\underline{\boldsymbol{X}}_2 = [\![\boxed{\boldsymbol{A}_3}, \boldsymbol{A}_4, \boxed{\boldsymbol{A}_5}]\!]$ $\quad$ $\underline{\boldsymbol{X}}_3 = [\![\boxed{\boldsymbol{A}_5}, \boldsymbol{A}_6, \boldsymbol{A}_7]\!]$

Applications of Coupled Tensor Decomposition

- **Joint PMF Learning.**
  - Joint PMF estimation is of great interest in statistical learning applications such as classification, recommender systems etc.
  - In general, joint PMF learning is a hard problem due to the very high dimensionality.
    * For e.g, if we have K random variables $Z_1, \ldots, Z_K$ each taking I_k values, then joint PMF $\Pr(Z_1 = i_1, \ldots, Z_K = i_K)$ is an estimation of $\prod_{k=1}^{K} I_k$ parameters.
  - In practice, it is impossible to directly estimate $\Pr(Z_1 = i_1, \ldots, Z_K = i_K)$ from sample averaging when K is large.
    * For large K, the probability of encountering any particular joint appearance of all the random variables is very negligible.

Joint PMF Learning

- Joint PMF $\Pr(Z_1 = i_1, \ldots, Z_K = i_K)$, where Z_k can take I_k different values can be represented as a Kth-order tensor $\underline{\boldsymbol{X}} \in \mathbb{R}^{I_1 \times \cdots \times I_K}$ with

$$\underline{\boldsymbol{X}}(i_1, \ldots, i_K) = \Pr(Z_1 = i_1, \ldots, Z_K = i_K).$$

- Since every tensor has a CPD representation, the joint PMF tensor $\underline{\boldsymbol{X}}$ can be written as [Kargas, et al. 2018],

$$\underline{\boldsymbol{X}}(i_1, \ldots, i_K) = \sum_{f=1}^{F} \prod_{k=1}^{K} \boldsymbol{\lambda}(f) \boldsymbol{A}_k(i_k, f), \quad \underline{\boldsymbol{X}} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K]\!].$$

where $\boldsymbol{\lambda} \in \mathbb{R}^F$, $\mathbf{1}^\top \boldsymbol{\lambda} = 1$, $\boldsymbol{\lambda} \geq \mathbf{0}$, $\boldsymbol{A}_k \in \mathbb{R}^{I_k \times F}$, $\mathbf{1}^\top \boldsymbol{A}_k = \mathbf{1}$ and $\boldsymbol{A}_k \geq \mathbf{0}$.

- Interestingly, CPD representation can be considered as naive Bayesian model w.r.t a latent random variable H,
  - $\boldsymbol{\lambda}$ can be the prior probability vector with $\boldsymbol{\lambda}(f) = \Pr(H = f)$,
  - $\boldsymbol{A}_k$ can be the conditional PMF matrix with $\boldsymbol{A}_k(i_k, f) = \Pr(Z_k = i_k | H = f)$.

Joint PMF Learning from Lower-order Marginals

- If we can estimate the latent factors $\lambda, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K$, the complete joint PMF matrix $\underline{\boldsymbol{X}}$ can be reconstructed.
  - If the tensor is of low rank, then the number parameters to be estimated is only around $\sum_{k=1}^{K} I_k F$.

- **How do we estimate the latent factors?**

Joint PMF Learning from Lower-order Marginals

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  - If the tensor is of low rank, then the number parameters to be estimated is only around $\sum_{k=1}^{K} I_k F$.
- **How do we estimate the latent factors?**
  - Lower-order marginals of two or three random variables are much easier to estimate and is related to the joint PMF as,

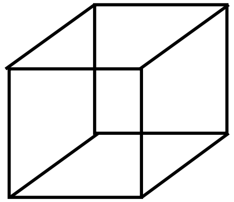
$$\mathsf{Pr}(Z_\ell = i_\ell, Z_m = i_m, Z_n = i_n) = \sum_{i_k \neq i_\ell, i_m, i_n} \sum_{i_k=1}^{I_k} \mathsf{Pr}(Z_1 = i_1, \ldots, Z_K = i_K)$$

    where $\mathsf{Pr}(Z_\ell = i_\ell, Z_m = i_m, Z_n = i_n)$ is the third-order marginal PMF.
  - [Kargas, et al. 2018] showed that given the joint PMFs of three random variables, the joint PMF of all the random variables can be provably recovered under mild conditions.

Joint PMF Learning from Lower-order Marginals

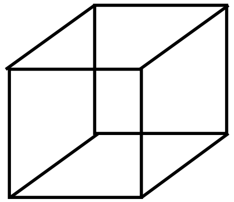
Marginalization | **Coupled Tensor Decomposition** | Reconstruction

$\Pr(Z_1 = i_1, \ldots, Z_7 = i_7)$



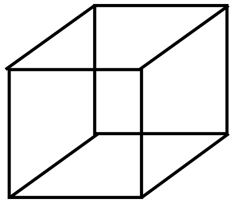
$\underline{\boldsymbol{X}}_{1,2,3} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3]\!]$

$\underline{\boldsymbol{X}}_{1,2,3}(i_1, i_2, i_3) = \Pr(Z_1 = i_1, Z_2 = i_2, Z_3 = i_3)$



$\underline{\boldsymbol{X}}_{3,4,5} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_3, \boldsymbol{A}_4, \boldsymbol{A}_5]\!]$

$\underline{\boldsymbol{X}}_{3,4,5}(i_3, i_4, i_5) = \Pr(Z_3 = i_3, Z_4 = i_4, Z_5 = i_5)$



$\underline{\boldsymbol{X}}_{5,6,7} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_5, \boldsymbol{A}_6, \boldsymbol{A}_7]\!]$

$\underline{\boldsymbol{X}}_{5,6,7}(i_5, i_6, i_7) = \Pr(Z_5 = i_5, Z_6 = i_6, Z_7 = i_7)$

$\hat{\Pr}(Z_1 = i_1, \ldots, Z_7 = i_7)$

$\underline{\boldsymbol{X}} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_7]\!]$

Coupled Tensor Decomposition

- Joint PMF can be learned via coupled tensor decomposition of the third order marginals using the below optimization problem [Kargas, et al. 2018],

$$\begin{aligned} \underset{\{\boldsymbol{A}_k\}_{k=1}^{K}, \boldsymbol{\lambda}}{\text{minimize}} \; & \sum_{\ell=1}^{K} \sum_{m=\ell+1}^{K} \sum_{n=m+1}^{K} \left\| \underline{\boldsymbol{X}}_{\ell,m,n} - [\![\boldsymbol{\lambda}, \boldsymbol{A}_\ell, \boldsymbol{A}_m, \boldsymbol{A}_n]\!] \right\|_F^2 \\ \text{subject to } & \mathbf{1}^\top \boldsymbol{A}_k = \mathbf{1}^\top, \; \boldsymbol{A}_k \geq \mathbf{0}, \; \forall k \\ & \mathbf{1}^\top \boldsymbol{\lambda} = 1, \; \boldsymbol{\lambda} \geq \mathbf{0}. \end{aligned}$$

- [Traganitis, et al. 2018] proposed a similar model and coupled tensor decomposition formulation for another popular statistical learning, ie. **crowsourcing**.

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\text{subject to } & \mathbf{1}^\top \boldsymbol{A}_k = \mathbf{1}^\top, \; \boldsymbol{A}_k \geq \mathbf{0}, \; \forall k \\
& \mathbf{1}^\top \boldsymbol{\lambda} = 1, \; \boldsymbol{\lambda} \geq \mathbf{0}.
\end{aligned}
$$

- [Traganitis, et al. 2018] proposed a similar model and coupled tensor decomposition formulation for another popular statistical learning, ie. **crowsourcing**.

How do we design an algorithm that can efficiently handle very large number of coupled tensors?

Existing Approaches

- The algorithms proposed in [Kargas, et al. 2018,Traganitis, et al. 2018] handling coupled tensor decomposition has the following features
  - Cyclic updates for each of $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \boldsymbol{\lambda}$
  - In each update of $\boldsymbol{A}_k$, solving a subproblem using unfolded version of $\underline{\boldsymbol{X}}_{\ell,m,n}$,

$$\underset{\substack{\mathbf{1}^\top \boldsymbol{A}_k = \mathbf{1}^\top \\ \boldsymbol{A}_k \geq \mathbf{0}}}{\text{minimize}} \sum_{m \neq k} \sum_{\substack{n \neq k \\ n > m}} \|\boldsymbol{X}_{k,m,n}^{(1)} - (\boldsymbol{A}_n \odot \boldsymbol{A}_m)\boldsymbol{D}\boldsymbol{A}_k^\top\|_F^2, \tag{1}$$

  - For e.g, for updating $\boldsymbol{A}_1$, (1) may be written as,

$$\underset{\substack{\mathbf{1}^\top \boldsymbol{A}_1 = \mathbf{1}^\top \\ \boldsymbol{A}_1 \geq \mathbf{0}}}{\text{minimize}} \left\| \begin{bmatrix} \boldsymbol{X}_{1,2,3}^{(1)} \\ \boldsymbol{X}_{1,2,4}^{(1)} \\ \vdots \\ \boldsymbol{X}_{1,2,K}^{(1)} \end{bmatrix} - \begin{bmatrix} (\boldsymbol{A}_3 \odot \boldsymbol{A}_2)\boldsymbol{D} \\ (\boldsymbol{A}_4 \odot \boldsymbol{A}_2)\boldsymbol{D} \\ \vdots \\ (\boldsymbol{A}_K \odot \boldsymbol{A}_2)\boldsymbol{D} \end{bmatrix} \boldsymbol{A}_1^\top \right\|_F^2$$

  - Using an ADMM algorithm to handle the subproblems.

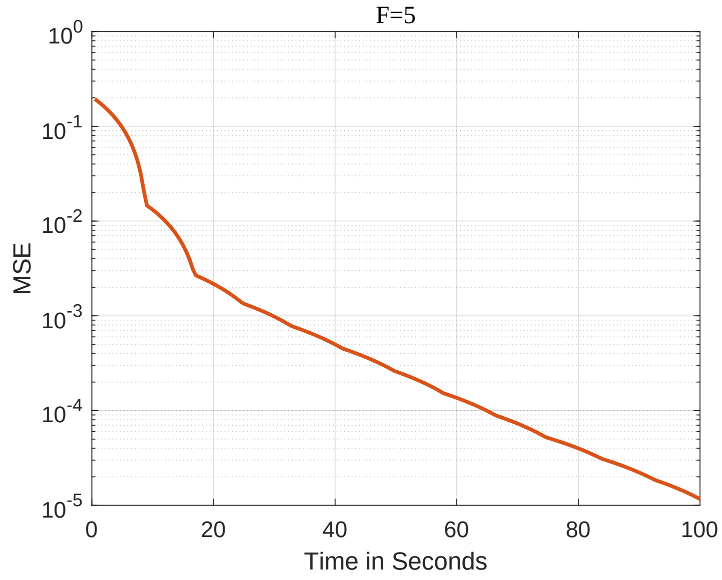
Existing Approaches

- **Challenges:**
  - Very large matrix-matrix product operation is to be performed in each step which substantially worsens the complexity.
  - This way, each step needs $O\left(\binom{K}{2} I_k I_m I_n F\right)$ flops $\implies O(I^5)$ if $K \approx I_\ell = I$.

Existing Approaches

- **Challenges:**
  - Very large matrix-matrix product operation is to be performed in each step which substatntially worsens the complexity.
  - This way, each step needs $O\left(\binom{K}{2} I_k I_m I_n F\right)$ flops $\implies O(I^5)$ if $K \approx I_\ell = I$.

Huge per-iteration complexity.



MSE of the latent factors when $K = 20, I_k = I = 15, F = 5$

Stochastic Optmization

- **Goal:** Accelerate coupled tensor decomposition using stochastic optimization.
- **How do we achieve this?**

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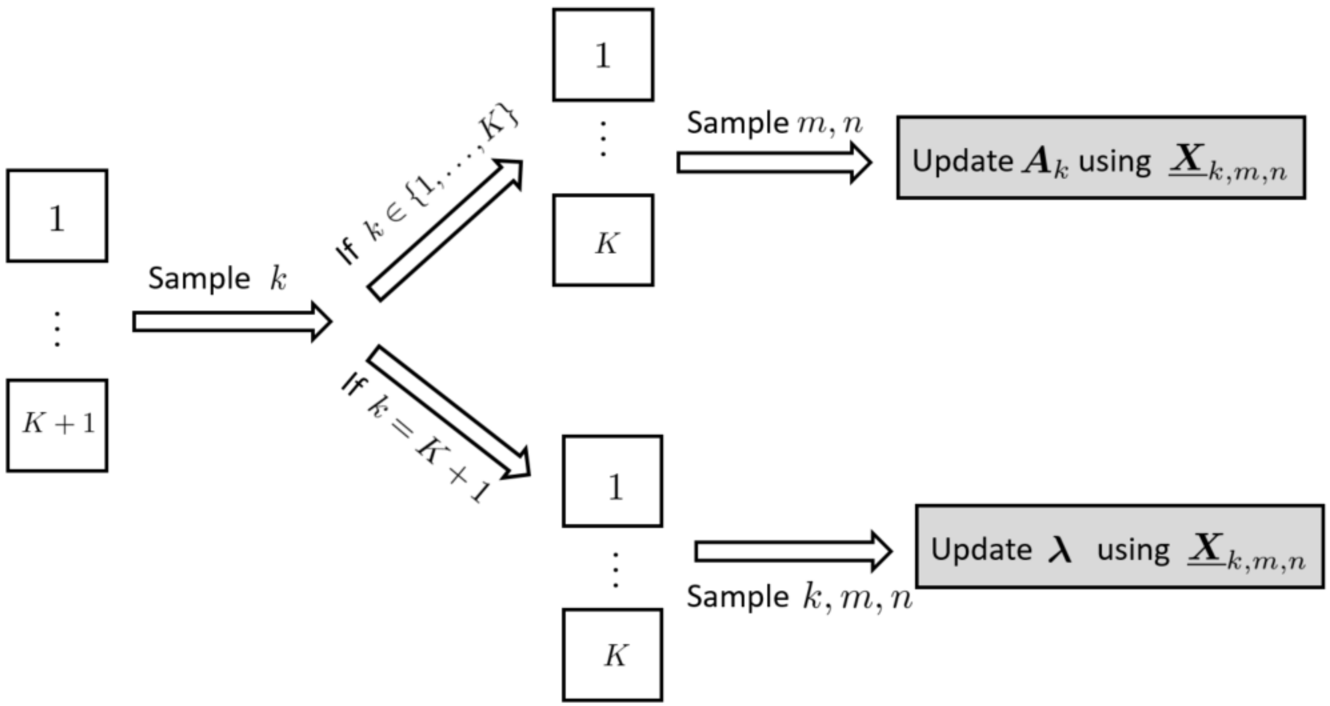
- **How do we achieve this?**

- An example of an existing idea used in [Vervliet, et al. 2016] is as below
  - Randomly sample part of the tensor, ie., a subtensor from $\underline{\boldsymbol{X}}$.
  - Then apply CPD to the sampled subtensor and update the latent factors.

- The idea of random sampling can be utilized in our case also, but
  - Do we need to run a complete CPD on the sampled tensor?
  - How do we ensure convergence of the algorithm involving random sampling?

Proposed Algorithm

- **Sampling strategy at each iteration t:**
  - Sample a block variable to update from $k \in \{1, \ldots, K, K+1\}$
  - Then, sample m, n from $\{1, \ldots, k-1, k+1, \ldots, K\}$ to update $\boldsymbol{A}_k$ or λ.



Proposed Algorithm

- **Update strategy at each iteration t:**
  - Assign

$$\boldsymbol{A}_k^{(t+1)} \leftarrow \mathsf{Proj}\left(\boldsymbol{A}_k^{(t)} - \alpha^{(t)}\boldsymbol{G}_k^{(t)}\right).$$

    where,

    * Stochastic Gradient, $\boldsymbol{G}_k^{(t)} = \boldsymbol{A}_k^{(t)}\boldsymbol{V}_k - (\boldsymbol{X}_{k,m,n}^{(1)})^\top \boldsymbol{H}_k$,
    * $\boldsymbol{H}_k = (\boldsymbol{A}_n \odot \boldsymbol{A}_m)\boldsymbol{D}$,
    * $\boldsymbol{V}_k = (\boldsymbol{\lambda}\boldsymbol{\lambda}^\top) \circledast (\boldsymbol{A}_n^\top \boldsymbol{A}_n) \circledast (\boldsymbol{A}_m^\top \boldsymbol{A}_m)$
    * Proj($\mathbb{Z}$) projects the columns of $\mathbb{Z}$ onto the probability simplex.
  - We also let $\boldsymbol{A}_j^{(t+1)} \leftarrow \boldsymbol{A}_j^{(t)}$, $\forall j \neq k$ and $\boldsymbol{\lambda}^{(t+1)} \leftarrow \boldsymbol{\lambda}^{(t)}$.
  - $\boldsymbol{\lambda}$ can also be updated in similar fashion.

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$$\boldsymbol{A}_k^{(t+1)} \leftarrow \text{Proj}\left(\boldsymbol{A}_k^{(t)} - \alpha^{(t)}\boldsymbol{G}_k^{(t)}\right). \Longleftarrow \textbf{Stochastic Proximal Gradient (SPG)}$$

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Features of the Proposed Algorithm

- **Favourable run-time for large scale problems.**
  - Lightweight algorithm compared to [Kargas, et al. 2018, Traganitis, et al. 2018] since update step uses only a single tensor $\underline{\boldsymbol{X}}_{k,m,n}$.
  - We do not have very large matrix-matrix product operation in each iteration.

- **Constraints to $\boldsymbol{A}_k$ or $\boldsymbol{\lambda}$ can be applied**
  - Each sampled $\underline{\boldsymbol{X}}_{k,m,n}$ contains information about entire $\boldsymbol{A}_k$ which makes applying constraints possible.

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- **What are the convergence gurantees?**

Convergence of the Proposed Algorithm

Proposition 1: Let,

$$f(\boldsymbol{\theta}) = \underset{\substack{\mathbf{1}^\top \boldsymbol{A}_k = \mathbf{1}^\top, \boldsymbol{A}_k \geq \mathbf{0} \\ \mathbf{1}^\top \boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \geq \mathbf{0}}}{\text{minimize}} \sum_{\ell=1}^{K} \sum_{m=\ell+1}^{K} \sum_{n=m+1}^{K} \left\| \underline{\boldsymbol{X}}_{\ell,m,n} - [\![\boldsymbol{\lambda}, \boldsymbol{A}_\ell, \boldsymbol{A}_m, \boldsymbol{A}_n]\!] \right\|_F^2$$

where $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^\top, \ldots, \boldsymbol{\theta}_K^\top]^\top$, $\boldsymbol{\theta}_k = \text{vec}(\boldsymbol{A}_k)$ for $k = 1, \ldots, K$ and $\boldsymbol{\theta}_{K+1} = \boldsymbol{\lambda}$. Let J_k denote the number of available tensors whose mode-1 factor is $\boldsymbol{A}_k$. Also let $\mathcal{B}^{(t)}$ be the filtration up to iteration $t-1$. Then, by uniform sampling of the tensors, the gradient computed at iteration t, $\boldsymbol{G}_k^{(t)}$ satisfies

$$\overline{\boldsymbol{G}}_k^{(t)} = \mathbb{E}\left[\boldsymbol{G}_k^{(t)} \mid \mathcal{B}^{(t)}\right] = C_k \nabla_{\boldsymbol{\theta}_k} f(\boldsymbol{\theta}), \ \forall k$$

where $C_k > 0$ is a certain constant.

Convergence Properties

- Proposition states that $\overline{\boldsymbol{G}}_k^{(t)}$ is a scaled version of the gradient of the objective function $f(\boldsymbol{\theta})$ taken w.r.t. ${\boldsymbol{A}_k}^{(t)}$.

- Two-stage sampling strategy results in SPG of $f(\boldsymbol{\theta})$ using an unbiased stochastic oracle $g^{(t)} = [\mathrm{vec}(\boldsymbol{G}_1^{(t)})^\top, \ldots, \mathrm{vec}(\boldsymbol{G}_{K+1}^{(t))})^\top]^\top$ in each iteration t.
  - all the convergence properties of the single-block SPG algorithm hold for the proposed algorithm.

- Another key consideration is stepsize scheduling for $\alpha^{(t)}$
  - SPG normally works under the Robbins-Monroe rule, i.e., $\sum_{t=0}^{\infty} \alpha^{(t)} = \infty$ and $\sum_{t=0}^{\infty} (\alpha^{(t)})^2 < \infty$.
  - In this work, we use the $Adagrad$ rule as proposed in [Fu, et al. 2018, Duchi, et al. 2011].

Experiments - Synthetic Data

- We consider a joint PMF recovery problem to the proposed method and the baseline `LS-BCD` [Kargas, et al. 2018].

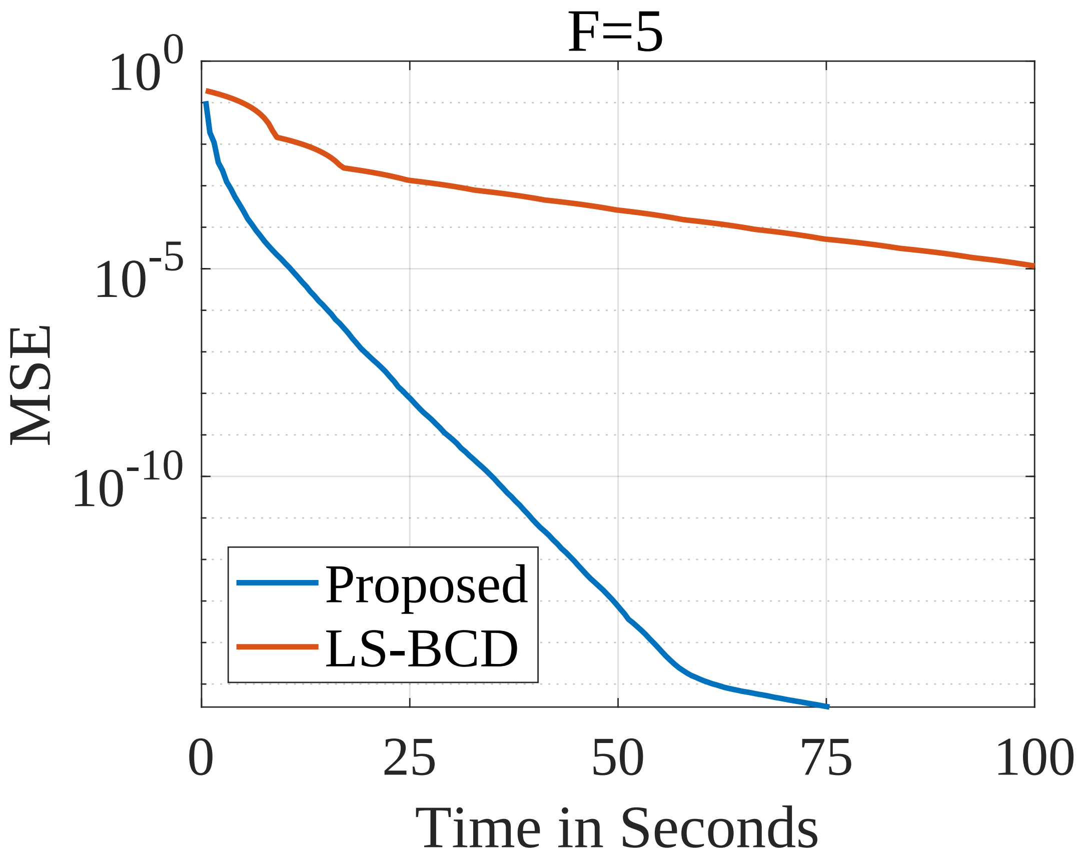
- We take $K = 20$ random variables with each variable taking $I_k = 15$ discrete values.

- The rank of the joint PMF tensor is set to different values $F \in \{5, 10\}$

- The columns of true latent factors $\boldsymbol{A}_k \in \mathbb{R}^{I_k \times F}$ and $\boldsymbol{\lambda} \in \mathbb{R}^F$ are drawn from the probability simplex uniformly at random.

- The third order statistics of the random variables $\underline{\boldsymbol{X}}_{\ell,m,n} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_\ell, \boldsymbol{A}_m, \boldsymbol{A}_n]\!]$, $\forall \ell, m, n \in [K]$ are generated using the true latent factors and used for estimation.

Synthetic Data Experiments



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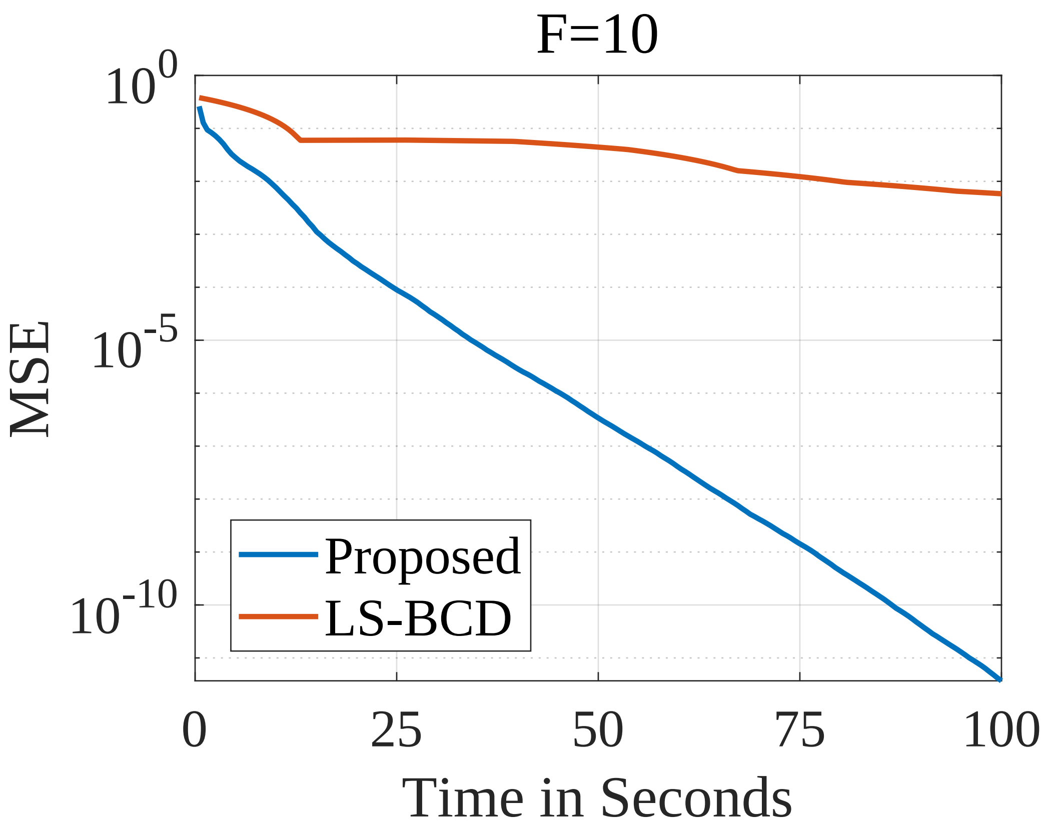

- The proposed algorithm outperforms the deterministic BCD algorithm `LS-BCD` in both accuracy and runtime by very large margin.

Real Data Experiments - Classification

- In this case, different UCI datasets[1] are used to evaluate the classification performance using the proposed method and the baselines `LS-BCD` [Kargas, et al. 2018] and `KL-BCD` [Kargas, et al. 2019]

- For each dataset, we run 10 Monte Carlo simulations by randomly partitioning the dataset into training, validation and testing sets.

- Using the training dataset, $\underline{\boldsymbol{X}}_{\ell,m,m}$ are estimated via counting the co-occurrences of the values taken by features ℓ, m and n.

- For each dataset, F is chosen by observing classification accuracy on the validation set.

- After identifying the parameters $\boldsymbol{A}_k$ and $\boldsymbol{\lambda}$, we use the *maximum a posteriori* (MAP) predictor to estimate the labels of the testing set.

[1] https://archive.ics.uci.edu/ml/datasets.html

Experiments-Real Data -Classification

Table 1: Real data-classification results using UCI dataset

	Misclassification(%)			Runtime(seconds)		
UCI Dataset (K, I_{avg}, F)	Proposed	LS-BCD	KL-BCD	Proposed	LS-BCD	KL-BCD
Nursery (9,4,15)	**0.086**	0.087	0.094	**2.98**	9.85	8.34
Car (7,4,15)	**0.097**	0.107	0.1068	**4.29**	14.79	7.725
Adult (15,14, 15)	**0.187**	0.247	†	**6.57**	48.49	†
Connect4 (22,7,15)	**0.338**	0.363	0.356	**6.54**	52.47	389.22
Credit (15,10,10)	**0.189**	0.347	0.254	**5.22**	40.02	30.51
Heart (9,3,10)	**0.198**	0.213	0.2113	**2.02**	8.87	8.11
Mushroom (21,6,15)	**0.042**	0.043	0.043	**8.19**	69.95	378.67
Voters (17,2,15)	**0.045**	0.076	0.053	**3.87**	27.44	27.64

† means the algorithm does not converge in 500 sec. and the result is not meaningful.

- The proposed algorithm outperforms the baselines in terms of accuracy and enjoys favourable run-time.

Experiments-Real Data -Crowdsourcing

- We use $K = 10$ different classification algorithms from the MATLAB machine learning toolbox to serve as annotators.

- Using 20% of the available data samples, each annotator is trained. Then, we allow the annotators to label the unseen data samples with probability p.

- Setting $p < 1$ is equivalent to the practical scenario where not all data samples are annotated by an annotator.

- Once the annotator responses are available, we estimate the co-occurrences of the annotator responses ℓ, m and n to obatin $\underline{\boldsymbol{X}}_{\ell,m,n}$.

- We perform 10 trials to take the average of the results and in each trial, a randomly selected testing set is labeled by the annotators with probability p.

- In our experiments, we set $p = 0.2$ for all annotators.

Experiments-Real Data -Crowdsourcing

- The classification accuracy of the proposed method is compared with different crowdsourcing algorithms such as `LS-ADMM` [Traganitis, et al. 2018] and `S-D&S` [Zhang, et al. 2014].

Table 2: Real data-crowdsouring results using UCI dataset

	Misclassification(%)			Runtime(seconds)		
UCI Dataset (K,F)	`Proposed`	`LS-ADMM`	`S-D&S`	`Proposed`	`LS-ADMM`	`S-D&S`
Adult (10,2)	**0.182**	0.258	0.238	**0.19**	4.17	2.10
Connect4(10,3)	**0.273**	0.344	0.333	**0.72**	50.96	14.38
Credit (10,2)	**0.166**	0.175	**0.166**	**0.18**	0.45	1.49
Mushroom (10,2)	**0.061**	0.064	**0.061**	**0.18**	0.44	2.40

- The proposed algorithm is very competitive in both accuracy and runtime.

Conclusion

- In this work, we proposed a stochastic sampling and optimization strategy for coupled tensor decomposition tailored for statistical learning problems.
- The algorithm can handle a large number of latent factor-coupled tensors and can easily deal with a variety of constraints on the latent factors.
- The algorithm admits an interesting connection to the classic single-block stochastic proximal gradient scheme—thereby enjoying the same convergence properties.
- Simulations and real experiments showed that the proposed algorithm outperforms various existing algorithms devised for similar problems in both runtime and accuracy.

Thank You

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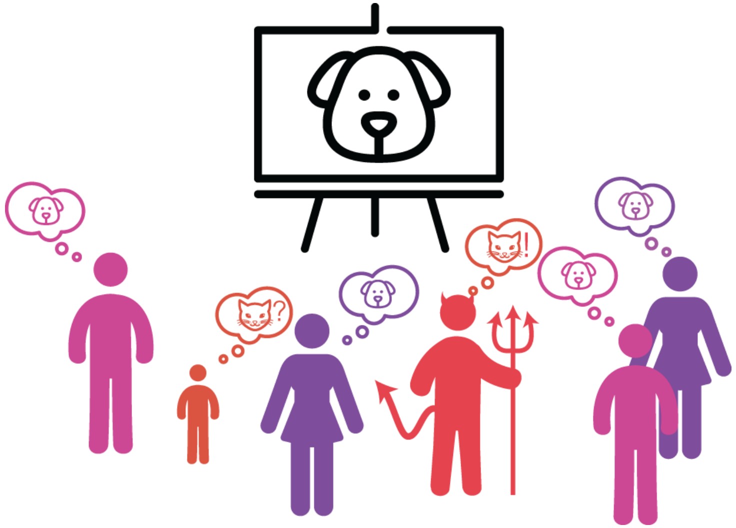
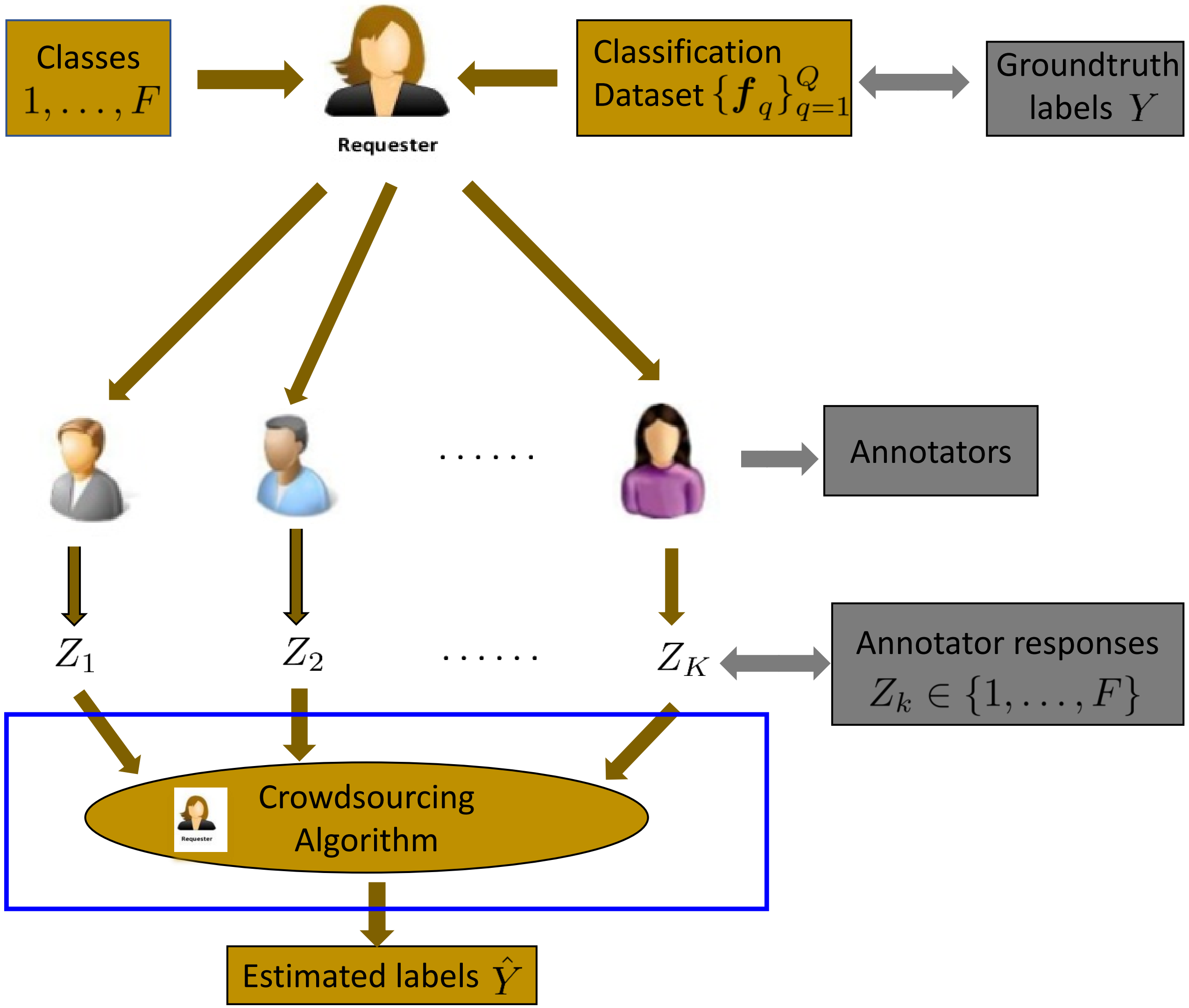
Applications of Coupled Tensor Decomposition

- **Crowdsourcing**
  - In machine learning, data labeling is oftentimes crowdsourced to multiple annotators for efficiency and robustness.



  - Since different annotators may create different labels for the same sample, an effective algorithm for result fusion is desired.

Crowdsourcing Dataflow



Crowdsourcing Model

- The classic model in crowdsourcing proposed by Dawid-Skene [1] is also a naive Bayesian model.

- ie., the responses of the annotators, $Z_1, \ldots, Z_K$ are conditionally independent given the true label Y,

$$\mathsf{Pr}(Z_1 = i_1, \ldots, Z_K = i_K) = \sum_{f=1}^{F} \mathsf{Pr}(Y = f) \prod_{k=1}^{K} \mathsf{Pr}(Z_k = i_k | Y = f),$$

  where $f \in \{1, \ldots, F\}$ represents the class label, and i_k denotes the response of the kth annotator.

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- The classic model in crowdsourcing proposed by Dawid-Skene [1] is also a naive Bayesian model.

- ie., the responses of the annotators, $Z_1, \ldots, Z_K$ are conditionally independent given the true label Y,

$$\underbrace{\mathsf{Pr}(Z_1 = i_1, \ldots, Z_K = i_K)}_{\underline{\boldsymbol{X}}(i_1,\ldots,i_K)} = \sum_{f=1}^{F} \underbrace{\mathsf{Pr}(Y = f)}_{\lambda(f)} \prod_{k=1}^{K} \underbrace{\mathsf{Pr}(Z_k = i_k | Y = f)}_{\boldsymbol{A}_k(i_k,f)},$$

where $f \in \{1, \ldots, F\}$ represents the class label, and i_k denotes the response of the kth annotator

Crowdsourcing Model

- Crowdsourcing problem admits the CPD model $\underline{\boldsymbol{X}} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K]\!]$. where the latent factors,
  - $\boldsymbol{\lambda}$ is called as the prior probability of true labels,
  - $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_K$ are called as the confusion matrices.

- The estimation of true labels from the latent factors can be done by MAP predictor [2].

- Here also, jointly observing the responses of all annotators is hard, thereby estimating the joint co-occurences ($\underline{\boldsymbol{X}}_{\ell,m,n}$) of three annotator responses is a viable solution.

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Coupled decomposition of all available tensors $\underline{\boldsymbol{X}}_{\ell,m,n} = [\![\boldsymbol{\lambda}, \boldsymbol{A}_\ell, \boldsymbol{A}_m, \boldsymbol{A}_n]\!]$ can solve the crowdsourcing problem.

References

[1] A. P. Dawid and A. M. Skene, "Maximum likelihood estimation of observer error-rates using the em algorithm," *Applied Statistics*, pp. 20–28, 1979.

[2] P. Traganitis, A. Pages-Zamora, and G. B. Giannakis, "Blind multiclass ensemble classification," *IEEE Trans. Signal Process.*, vol. 66, pp. 4737–4752, Jul 2018.